SYNOPSIS

Title: "A STUDY OF HOMOMORPHISMS GROUP IN RING THEORY"

a) WORK PLAN

We propose to investigate group rings, which are constructed as free modules over a ring and incorporate both ring-theoretic and group-theoretic structures. In this study, we view the group ring as a generalization of group theory, obtained by assigning a "weighting factor" from the ring to each element of the group. When the underlying ring is commutative, the resulting structure is known as a group algebra, which forms an algebra over that ring. The properties of group rings play an important role in the theory of group representations. Furthermore, we aim to extend the Bass–Milnor Theorem for abelian groups, showing that generalized Bass units reduce to the classical Bass units. We also consider the structure of epimorphic images, which may take the form of non-commutative division algebras or 2 x 2 matrix algebras over division algebras. In addition, we propose to extend the work of S. K. Singhal and Ritter on units in integral group rings. We construct a basis consisting of products of Bass units for a free abelian subgroup of finite index. Moreover, we extend the result showing that the group generated by these units contains a subgroup of finite index within the central units of the integral group ring ZG for finite strongly monomial groups G. Our work further generalizes several theorems, including the Bass–Milnor theorem for abelian groups. We also extend results previously established for abelian groups by employing the Wedderburn decomposition of the rational group algebra, assuming no exceptional components. The primary focus is to determine the automorphism group of U(ZG) when Gis a finitely generated abelian group.

b) DETAILS OF CHAPTALISATION

W. Kimmerle and Roggenkamp reformulated a Sylow-like theorem for V(ZG) by considering several variants. The weak Sylow theorem states that every finite p-subgroup of V(ZG) is isomorphic to some subgroup of G, while the strong Sylow theorem asserts that each finite p-subgroup of V(ZG) is even rationally conjugate to a subgroup of G. we derive the concerning Sylow-like results for integral group rings, M. A. Dokuchaev and S. O. Juriaans established a strong Sylow theorem for specific

classes of solvable groups. We propose to extend the generalized structure of torsion units in V(ZG), building upon the work of Marciniak and Angela Valenti.

We focus on extending the applications of Boolean rings, subrings, and both commutative and non-commutative ring structures. The entire research work is organized into four chapters, as outlined below.

Chapter – 1 Integral Group Ring and its representations

In this chapter, we examine integral group rings of metabelian groups, the structure of central units and idempotents in group algebras, and the integral isomorphism problem along with its classifications.

Chapter - 2 Algebraic structure of group rings and its applications

Abelian groups play a significant role in group theory and are equally important in the formulation of group-ring theory. This chapter also explores the isomorphic structure of Sylow subgroups.

Chapter - 3 Properties of commutative and non-commutative Group

A monomial non-commutative group ring is a formal structure built from a non-commutative group and a ring, typically using group elements as monomial basis elements. It combines group elements with ring coefficients, with multiplication defined distributively so that it preserves both the non-commutative group operation and the algebraic structure of the ring.

Chapter - 4 Structural formulation of Sylow group in Integral group ring

The term primitive structure focuses on the conjugacy relations, the existence of Sylow p-subgroups, their normality conditions, and their internal composition. In the integral group ring ZG, the finite subgroups of V(ZG) are studied using theoretical methods and character theory. The structure of these units is closely connected to the character table of G and the properties of cyclotomic fields.

c) HYPOTHESIS

We proposed to develop a general ring theory in the o-minimal setting culminating in a description of all the definable rings in an arbitrary o-minimal structure by A Conversano. We derive definably connected ring with non-trivial multiplication defines

an infinite field and it is essentially semi-algebraic. A surprisingly strong correspondence between definably connected rings and finite dimensional associative R-algebras is established and ideal of a definable unital ring is definable, from which it follows that every definable unital ring is Artinian and Noetherian. We propose to develop the fundamental concepts of monomial non-commutative group rings. The notions of commutative and non-commutative integral group rings were introduced by Sehgal, and we aim to present specific classifications of the algebraic structure of group rings. Additionally, we derive necessary and sufficient conditions for classifying those finite groups that possess a faithful exceptional 2×2 matrix ring component. We also investigate the commensurability properties of the various types of 2×2 matrix rings that appear as simple components in the Wedderburn decomposition of the rational group algebra of a finite group.

d) METHODOLOGY

We propose to introduce an enhanced formulation of the Sylow group theorem within the context of integral group rings, focusing on Sylow subgroups and demonstrating that the properties of primitive structures support existing constructions for finitely generated nilpotent groups.

Torsion units in integral group rings, as studied in Reine Angew., show that idempotents are sufficient to describe the primitive central idempotents of an abelian-by-supersolvable group. A supersolvable group is one that possesses a normal series whose factors are cyclic, while a group is abelian-by-supersolvable if it contains an abelian normal subgroup Asuch that G/A is supersolvable.

We further develop properties of transitive permutation groups to demonstrate that every transitive group of prime-power degree contains a transitive Sylow subgroup. The main focus lies in determining the isomorphism and normalization of these groups. Additionally, we examine the cardinal structure of Sylow p-subgroups and characterize them through homomorphisms, highlighting applications of p-Sylow subgroups in algebraic number theory.

This research project is entirely theoretical in nature, focusing on ring theory and its homomorphic properties. The central library of B. N. Mandal University, Madhepura, provides the primary resources needed for this work. If required, additional references

both recent and archival may be consulted at the National Library (Science Section) and the Indian Statistical Institute (ISI), Kolkata.

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